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Entropy of the Quantum Electromagnetic Field in Static, Spherically Symmetric Dilaton Black Holes

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The quantum correction to the entropy of four-dimensional nonextreme static, spherically symmetric dilaton black holes arising from electromagnetic fields is investigated by 't Hooft's "brick wall" model. The Garfinkle–Horowitz–Strominger, Gibbons–Maeda, and Garfinkle–Horne dilaton black holes are considered. It is shown that the one-loop quantum correction arising from the electromagnetic fields is exactly twice that due to a massless scalar field. The result agrees with that of the Schwarzschild and Reissner–Nordström black holes.

1. INTRODUCTION

Since Bekenstein and Hawking found that the black hole entropy is proportional to the event horizon by comparing black hole physics with thermodynamics, and since the discovery that black holes generate thermal radiation [1–3] much effort has been directed at the study of the quantum entropy of the black holes [4–22] in the hope that such study can shed light on the problem of obtaining a statistical meaning for the Bekenstein–Hawking entropy. 't Hooft [4] argued that the black hole entropy is identified with the quantum entropy arising from a thermal bath of quantum fields propagating outside the horizon. In order to eliminate the divergence which appears due to the infinite growth of the density of states close to the horizon, 't Hooft introduced a "brick wall" cutoff: a fixed boundary near the event horizon within the quantum field does not propagate and the Dirichlet boundary condition is imposed on the boundary. The "brick wall" model (BWM) has

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been successfully used in studies of the quantum entropy for many black holes [4, 6, 16, 17].

Most work on the quantum entropy of black holes is carried out using quantum scalar fields. The study of the quantum correction to black hole entropy due to the electromagnetic field has recently been carried out. By using the heat-kernel and proper-time regularization, Kabat [23] studied the entropy for the electromagnetic field case in Rindler space. He obtained an unexpected surface term which corresponds to particle paths beginning and ending at the event horizon. This term gives a negative contribution to the entropy of the system and is large enough to make the total entropy negative at the equilibrium temperature. However, Iellici and Moretti [24, 25], using a local ζ-function regularization approach, proved that the surface term is gauge dependent in the four-dimensional case and therefore can be discarded. Recently, Cognola and Lecca [26] used the BWM to study the quantum entropy for electromagnetic fields in the Schwarzschild and Reissner–Nordström black hole spacetimes. They found that there is no such surface term in the BWM, and the leading term of the entropy for the electromagnetic fields is exactly twice that for a massless scalar field. However, the question of whether or not the relation for the Schwarzschild and Reissner–Nordström black holes obtained by Cognola and Lecca is valid for other black holes, especially for dilaton black holes, remains open. The purpose of this paper is to settle the question by studying the quantum entropy of the electromagnetic field in static dilaton black holes.

The paper is organized as follows: In Section 2, we first show that the electromagnetic field in the four-dimensional static dilaton black hole spacetimes can be expressed in term of a couple of scalar fields satisfying Klein–Gordon-like equations. We then get the total number of modes with energy less than E, and use it to calculate the free energy. The quantum entropies are then obtained by the variation of the free energy with respect to inverse temperature. In last section, the results are compared with those due to the scalar field, and a discussion is presented.

2. ENTROPY OF THE QUANTUM ELECTROMAGNETIC FIELD IN STATIC, SPHERICALLY SYMMETRIC DILATON BLACK HOLES

The line element for four-dimensional static, spherically symmetric dilaton black holes can be expressed as

$$ds^2 = g_{tt}dt^2 + g_{rr}dr^2 + R_1(d\theta^2 + \sin^2\theta \, d\varphi^2) \tag{1}$$

where g_{tt}, g_{rr}, and R_1 are functions of the coordinate r only. In this paper we focus on the Garfinkle–Horowitz–Strominger, Gibbons–Maeda, and

Garfinkle–Horne dilaton black holes. For these black holes we have $g_{tt} = -1/g_{rr} = -g^{rr} \equiv -g$.

After introducing new coordinates $z = \sin\theta\, e^{i\alpha\varphi}/(1 - \cos\theta)$ and $\bar{z} = \sin\theta\, e^{-i\alpha\varphi}/(1 - \cos\theta)$, we rewrite the line element (1) as

$$ds^2 = -g\, dt^2 + \frac{1}{g}\, dr^2 + g_{z\bar{z}}\, dz\, d\bar{z} \tag{2}$$

with

$$g_{z\bar{z}} = \frac{2R_1}{(1 + z\bar{z})^2} \tag{3}$$

We now try to find a quantum entropy expression for electromagnetic fields in thermal equilibrium at temperature $1/\beta$ in static, spherically symmetric dilaton black holes. The partition function is

$$Z = \sum_{n_q} \exp[-\beta(E_q)n_q] \tag{4}$$

where q denotes the quantum state of the field with energy E_q. The free energy is given by [4]

$$\begin{aligned} F &= -\frac{1}{\beta}\sum_{\nu} \ln(1 - e^{-\beta E}) \\ &= \frac{1}{\beta}\int_0^\infty d\nu(E)\, \ln\,(1 - e^{-\beta E}) \\ &= -\frac{1}{\beta}\int_0^\infty \frac{\nu(E)}{e^{\beta E} - 1}\, dE \end{aligned} \tag{5}$$

where $\nu(E) \equiv \pi N$, and N is the total number of waves with energy not exceeding E. In the third line of Eq. (5) we have integrated by parts. In the following we first look for an expression for $\nu(E)$.

Substituting the electromagnetic potential A_μ into the Maxwell equation

$$\nabla_i F^{ij} = 0 \tag{6}$$

we obtain

$$\Box A_k - \nabla_i \nabla_k A^i = \Box A_k - \nabla_k \nabla_i A^i - R_{ki}A^i = 0 \tag{7}$$

where $\Box = g^{ij}\, \nabla_i\, \nabla_j$ is the D'Alembertian operator, and R_{kj} is the Ricci tensor. By using the expression for the Ricci tensor we have [26]

$$LA_i = A_j\partial_i\, (g^{kl}\Gamma^j_{kl}) + 2g^{kl}\Gamma^j_{ki}\partial_l A_j + \partial_i\, \nabla_j A^j \tag{8}$$

with

$$L = \frac{1}{\sqrt{-g}}\,\partial_i\sqrt{-g}g^{ij}\,\partial_j \tag{9}$$

Substituting the metric (2) into (8), we obtain

$$LA_t = \frac{dg}{dr}\left(\frac{\partial A_t}{\partial r} - \frac{\partial A_r}{\partial t}\right) + \frac{\partial}{\partial t}\nabla_i A^i \tag{10}$$

$$\begin{aligned} LA_r = {} & -\left[\frac{d^2 g}{dr^2} + \frac{d}{dr}\left(g\,\frac{d\ln R_1}{dr}\right)\right] A_r + g^{z\bar{z}}\,\frac{d\ln R_1}{dr}\left(\frac{\partial A_z}{\partial \bar{z}} + \frac{\partial A_{\bar{z}}}{\partial z}\right) \\ & - \frac{1}{g^2}\,\frac{dg}{dr}\,\frac{\partial A_t}{\partial t} - \frac{dg}{dr}\,\frac{\partial A_r}{\partial r} + \frac{\partial}{\partial r}\nabla_i A^i \end{aligned} \tag{11}$$

$$LA_z = -2\,\frac{\partial g^{z\bar{z}}}{\partial z}\,\frac{\partial A_z}{\partial \bar{z}} + g\,\frac{d\ln R_1}{dr}\left(\frac{\partial A_z}{\partial r} - \frac{\partial A_r}{\partial z}\right) + \frac{\partial}{\partial z}\nabla_i A^i \tag{12}$$

$$LA_{\bar{z}} = -2\,\frac{\partial g^{z\bar{z}}}{\partial \bar{z}}\,\frac{\partial A_{\bar{z}}}{\partial z} + g\,\frac{d\ln R_1}{dr}\left(\frac{\partial A_{\bar{z}}}{\partial r} - \frac{\partial A_r}{\partial \bar{z}}\right) + \frac{\partial}{\partial \bar{z}}\nabla_i A^i \tag{13}$$

In order to select the physical degrees of freedom, we fix the gauge condition $A_t = 0$. Then Eq. (10) gives a constraint condition

$$\nabla_i A^i + \frac{dg_{tt}}{dr}\,A_r = -\frac{dg}{dr}\,A_r + \frac{1}{R_1}\,\frac{\partial}{\partial r}\,(R_1 g A_r) + g^{z\bar{z}}\left(\frac{\partial A_{\bar{z}}}{\partial z} + \frac{\partial A_z}{\partial \bar{z}}\right) = 0 \tag{14}$$

Using the gauge and the constraint conditions, we can express Eqs. (11)–(13) as

$$LA_r = -\frac{1}{R_1}\,\frac{\partial}{\partial r}\left(g\,\frac{dR_1}{dr}\,A_r\right) \tag{15}$$

$$\left(L + 2\,\frac{\partial g^{z\bar{z}}}{\partial z}\,\frac{\partial}{\partial \bar{z}}\right)A_z = g\,\frac{d\ln R_1}{dr}\,\frac{\partial A_z}{\partial r} + g\,\frac{d}{dr}\left[\ln\left(\frac{-g}{R_1}\right)\right]\frac{\partial A_r}{\partial z} \tag{16}$$

$$\left(L + 2\,\frac{\partial g^{z\bar{z}}}{\partial \bar{z}}\,\frac{\partial}{\partial z}\right)A_{\bar{z}} = g\,\frac{d\ln R_1}{dr}\,\frac{\partial A_{\bar{z}}}{\partial r} + g\,\frac{d}{dr}\left[\ln\left(\frac{-g}{R_1}\right)\right]\frac{\partial A_r}{\partial \bar{z}} \tag{17}$$

Form Eqs. (15)–(14) we obtain two classes of independent electromagnetic potentials:

$$A_\mu^{\mathrm{I}} = \left(0,\, 0,\, \frac{1}{\sqrt{2l(l+1)\omega}} \frac{\partial \Phi_1}{\partial z},\, \frac{-1}{\sqrt{2l(l+1)\omega}} \frac{\partial \Phi_1}{\partial \bar{z}}\right) \tag{18}$$

$$A_\mu^{\mathrm{II}} = \left(0,\, \sqrt{\frac{l(l+1)}{2\omega^3}} \frac{\Phi_2}{R_1},\, \frac{g}{\sqrt{2l(l+1)\omega^3}} \frac{\partial^2 \Phi_2}{\partial z \partial r},\, \frac{g}{\sqrt{2l(l+1)\omega^3}} \frac{\partial^2 \Phi_2}{\partial \bar{z} \partial r}\right) \tag{19}$$

which show that A_μ^{I} and A_μ^{II} depend on a couple of scalar functions Φ_1 and Φ_2, respectively. Since both Φ_1 and Φ_2 satisfy the Klein–Gordon-like equation

$$\Box \Phi_i = g \frac{d \ln R_1}{dr} \frac{\partial \Phi_i}{\partial r}, \qquad i = 1, 2 \tag{20}$$

where $\Phi_i(t, r, z, \bar{z}) = e^{-\omega t} f_i(r) Y_l^m(z, \bar{z})$, hereafter we will drop the subscript on the Φ_i. It is interesting to note that the solutions (18) and (19) form a set of orthonormal eigenfunctions with respect to the scalar product.

We introduce the brick-wall boundary condition in which the wave function is cut off just outside the event horizon, i.e., $\Phi = 0$ at $r = r_H + h$ (h is a small, positive quantity and signifies an ultraviolet cutoff) and an infrared cutoff, $\Phi = 0$, at $r = L$ ($L >> r_H$). When we employ the WKB approximation and insert the metric (2) into Eq. (20), after discussion as in ref. 18, we find

$$\begin{aligned}
\nu(E) &= \sum_l (2l+1) n_r \\
&= \frac{1}{\pi} \int (2l+1)\, dl \int_{r_H+h}^{L} \frac{1}{g} \sqrt{E^2 - \frac{g l(l+1)}{R_1}}\, dr \\
&= \frac{2E^3}{3\pi} \left[R_1 \left(\frac{dg}{dr}\right)^{-2} \right]_{r_H} \frac{1}{h} \\
&\quad + \frac{2E^3}{3\pi} \left[\left(\frac{dg}{dr}\right)^{-3} \left(\frac{dg}{dr} \frac{dR_1}{dr} - R_1 \frac{d^2 g}{dr^2} \right) \right]_{r_H} \ln \frac{L}{h}
\end{aligned} \tag{21}$$

Since there are two independent scalar fields both satisfying Eq. (20), substituting Eq. (21) into (5), we find that the free energy at inverse temperature β can be expressed as

$$\begin{aligned}
\mathrm{B}F = -\frac{4\pi^3}{45\beta^3} \Bigg\{ & \left[R_1 \left(\frac{dg}{dr}\right)^{-2} \right]_{r_H} \frac{1}{h} \\
& + \left[\left(\frac{dg}{dr}\right)^{-3} \left(\frac{dg}{dr} \frac{dR_1}{dr} - R_1 \frac{d^2 g}{dr^2} \right) \right]_{r_H} \ln \frac{L}{h} \Bigg\}
\end{aligned} \tag{22}$$

If we set $\delta^2 = 2\epsilon^2/15$ and $\Lambda^2 = L\epsilon/h$ as in refs. 17 and 19 (where $\delta = \int_{r_H}^{r_H+h}\sqrt{g_{rr}}\,dr = 2\sqrt{(dg/dr)_{r_H}^{-1}h}$ is the proper distance from the horizon r_H to $r_H + h$, ϵ is the ultraviolet cutoff, and Λ is the infrared cutoff of Solodukhin [27, 28]) and note the Hawking inverse temperature, $\beta_H = [4\pi(dg/dr)^{-1}]_{r_H}$, we can rewrite Eq. (22) as

$$\beta F = -\frac{1}{24}\frac{R_1}{\epsilon^2}\left(\frac{\beta_H}{\beta}\right)^3 - \frac{R_1}{360}\left(\frac{\beta_H}{\beta}\right)^3\left(\frac{1}{R_1}\frac{dg}{dr}\frac{dR_1}{dr} - \frac{d^2g}{dr^2}\right)_{r_H}\ln\frac{\Lambda}{\epsilon} \tag{23}$$

The quantum correction to the entropy of the static dilaton black holes is given by

$$\begin{aligned} S^q_{\text{em}} &= \left(\beta^2\frac{\partial F}{\partial\beta}\right)_{\beta_H} \\ &= \frac{A_\Sigma}{24\pi\epsilon^2} - \frac{A_\Sigma}{360\pi}\left(\frac{d^2g}{dr^2} - \frac{1}{R_1}\frac{dg}{dr}\frac{dR_1}{dr}\right)_{r_H}\ln\frac{\Lambda}{\epsilon} \end{aligned} \tag{24}$$

In the above calculations, we ignored the contribution from the vacuum surrounding the system. We now consider some particular examples.

2.1. The Garfinkle–Horowitz–Strominger Dilatonic Black Hole

The metric of the Garfinkle–Horowitz–Strominger dilatonic black hole is given by [29]

$$\begin{aligned} ds^2 = -&\left(1 - \frac{2M}{r}\right)dt^2 + \frac{dr^2}{1 - 2M/r} \\ &+ r(r - a)(d\theta^2 + \sin^2\theta\, d\varphi^2) \end{aligned} \tag{25}$$

where $r_+ = 2M$ and $a = (Q^2/2M)e^{-2\phi_0}$. The parameters M and Q represent the mass and electric charge of the hole, respectively. From Eqs. (24) and (25) we know that the quantum correction to the entropy of the black hole is

$$S^q_{\text{GHS}} = \frac{A_H}{24\pi\epsilon^2} + \left(\frac{1}{90} + \frac{1}{30}\frac{A_H}{r_+\beta_H}\right)\ln\left(\frac{\Lambda}{\epsilon}\right) \tag{26}$$

where $A_H = 4\pi r_+(r_+ - a)$, $\beta_H = 4\pi r_+$, $\epsilon = \sqrt{30 r_+ h}$, and $\Lambda = \sqrt{30 r_+ L}$.

2.2. The Static Gibbons–Maeda Dilaton Black Hole

The static Gibbons–Maeda dilaton black hole is described by the metric [30]

$$ds^2 = -\frac{(r - r_+)(r - r_-)}{R^2}\,dt^2 + \frac{R^2\,dr^2}{(r - r_+)(r - r_-)} + R^2(d\theta^2 + \sin^2\theta\,d\varphi^2) \tag{27}$$

with

$$\begin{aligned} r_\pm &= M \pm \sqrt{M^2 + D^2 - P^2 - Q^2}, \\ D &= (P^2 - Q^2)/2M, \\ R^2 &= r^2 - D^2 \end{aligned} \tag{28}$$

The parameters Q and P represent electric charge and magnetic charge, respectively.

The quantum correction to the entropy in the background of the black hole can be obtained by using Eqs. (24) and (27), and is explicitly given by

$$S^q_{\mathrm{GM}} = \frac{A_H}{24\pi\epsilon^2} + \left(-\frac{1}{45} + \frac{1}{15}\,\frac{4\pi r_+}{\beta}\right)\ln\left(\frac{\Lambda}{\epsilon}\right) \tag{29}$$

where

$$A_H = 4\pi(r_+^2 - D^2), \qquad \beta_H = \frac{4\pi(r_+^2 - D^2)}{r_+ - r_-}$$
$$\epsilon = \sqrt{30h\,\frac{r_+^2 - D^2}{r_+ - r_-}}, \qquad \Lambda = \sqrt{30L\,\frac{r_+^2 - D^2}{r_+ - r_-}}$$

2.3. The Garfinkle–Horne Dilaton Black Hole

The Garfinkle–Horne dilaton black hole metric in the Einstein–Maxwell dilaton theory can be expressed as [29, 31]

$$\begin{aligned} ds^2 = &-\left(1 - \frac{r_+}{r}\right)\left(1 - \frac{r_-}{r}\right)^{(1-a^2)/(1+a^2)} dt^2 \\ &+ \left(1 - \frac{r_+}{r}\right)^{-1}\left(1 - \frac{r_-}{r}\right)^{(a^2-1)/(1+a^2)} dr^2 \\ &+ r^2\left(1 - \frac{r_-}{r}\right)^{2a^2/(1+a^2)} (d\theta^2 + \sin^2\theta\,d\varphi^2) \end{aligned} \tag{30}$$

with dilaton field

$$e^{2\Phi} = \left(1 - \frac{r_-}{r}\right)^{2a/(1+a^2)} e^{-2\Phi_0}$$

and Maxwell field $F = (Q/r^2)\, dt \wedge dr$. Here a is a coupling constant, and $r = r_+$ is the location of the event horizon. For $a = 0$, $r = r_-$ is the location of the inner Cauchy horizon; however, for $a > 0$ the surface $r = r_-$ is singular. The mass M and charge Q of the black hole are related to parameters r_+ and r_- by

$$2M = r_+ + \left(\frac{1 - a^2}{1 + a^2}\right) r_-, \qquad Q^2 = \frac{r_+ r_-}{1 + a^2}\, e^{2a\Phi_0} \tag{31}$$

We know from the metric (30) that the Hawking inverse temperature and the area of the event horizon can respectively be expressed as

$$\beta_H = \frac{2\pi}{\kappa} = \frac{4\pi r_+}{(1 - r_-/r_+)^{(1-a^2)/(1+a^2)}} \tag{32}$$

$$A_H = 4\pi r_+^2 \left(1 - \frac{r_-}{r_+}\right)^{2a^2/(1+a^2)} \tag{33}$$

Inserting the metric (30) into relations $\delta^2 = 2\epsilon^2/15$ and $\Lambda^2 = L\epsilon^2/h$, we get

$$\epsilon = \sqrt{\frac{15}{2}} \int_{r_+}^{r_+ + h} \sqrt{g_{rr}}\, dr = \sqrt{30h}\, r_+^{1/(1+a^2)} (r_+ - r_-)^{(a^2-1)/2(1+a^2)} \tag{34}$$

$$\Lambda = \sqrt{30L}\, r_+^{1/(1+a^2)} (r_+ - r_-)^{(a^2-1)/2(1+a^2)} \tag{35}$$

Then, the quantum correction (24) for the black hole can be written as

$$S^a_{\mathrm{GH}} = \frac{A_H}{24\pi\epsilon^2} + \frac{1}{45}\left(\frac{2a^2 - 1}{1 + a^2} + \frac{3}{1 + a^2}\frac{A_H}{r_+\beta}\right) \ln \frac{\Lambda}{\epsilon} \tag{36}$$

3. DISCUSSION AND CONCLUSION

By comparing results (26), (29), and (36) with Eqs. (34)–(36) in ref. 18, respectively, we know that for the Garfinkle–Horowitz–Strominger, Gibbons–Maeda, and Garfinkle–Horne dilaton black holes the quantum corrections to the entropies of the black holes arising from electromagnetic fields are exactly twice those due to massless scalar fields. The entropies (26), (29), and (36) consist of two parts as usual: a quadratically divergent term and a logarithmically divergent term. The quadratic part takes the geometric character $A_H/48\pi\epsilon^2$, which can be regarded as a renormalization of the gravitational constant $1/G_{\text{ren}} = 1/G + 1/(12\pi\epsilon^2)$. The logarithmically divergent term is not proportional to the horizon area. The term depends on the black hole characteristics, and therefore cannot be neglected as nonessential additive constants. The logarithmic divergence can be absorbed in the renormalization of the coupling constants by using a standard approach [32, 5, 10].

To summary, we first show that by imposing the gauge $A_t = 0$ the electromagnetic potential A_i in the static dilaton black holes can be expressed in term of two independent scalar fields. Then, by using 't Hooft's BWM the quantum corrections to the entropies of the static dilaton black holes arising from the. electromagnetic filed are obtained. The results show that the entropies are exactly twice those due a massless scalar field. The conclusion agrees with Cognola and Lecca's, which is obtained in the Schwarzschild and Reissner–Nordström black hole spacetimes. The renormalization of the quantum entropies is also discussed briefly.

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